Dark Matter on Galactic scales

Brief presentation of the challenges and prospects

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CONTENTS

I. INTRODUCTION

Dark matter (DM) refers to the various candidates for non-luminous matter in the universe whose effects have only been observed so far through the gravitational pull it exerts. Its identity and properties are still one of the biggest mysteries in modern physics. While direct detection of dark matter interacting with other forms of matter is lacking, its existence is supported by mounting evidence of its gravitational effects, accumulated over many decades.

The standard model of cosmology (ΛCDM) postulates the simplest form of dark matter: a single species, effectively collisionless and non-relativistic. This model has been remarkably successful in matching cosmological observations on large scales, from the background expansion history to the cosmic microwave background anisotropies and the linear growth of cosmic structures. However, on galactic scales a number of challenges have emerged and whether or not they can be resolved within the context of ΛCDM is still a subject of debate [1, 2].

In this paper, I will present the three main discrepancies between the cold dark matter (CDM) predictions and observations, namely (1) the core-cusp problem, (2) the missing satellite problem and (3) the low scatter from the baryonic scaling relations. It is important to emphasize here that these issues originally arose from the comparison of observations to theoretical predictions of CDM-only simulations[1]. Hence, as a way to resolve them within the

[1] For completeness, there are two additional well known discrepancies of CDM at the galactic scale: (1) the too big to fail problem (TBTF) and (2) the planes of satellite galaxies problem (PSP). I briefly mention the first one in sec.II B where I point out that under reasonable assumptions it essentially reduces to the core-cusp problem. On the other hand, the PSP refers to the peculiar spatial alignments and kinematic coherence of the orbits of the Milky Way satellites, which are extremely rare in CDM simulations. In short, it can be thought as an added level of complexity to the missing satellite problem, which concerns the distribution of these satellites rather than simply their population size. With the goal of keeping the discussion concise, I omit a detailed presentation of these two additional discrepancies. The interested reader may refer to [3, 4].

ΛCDM framework, a large part of the research in this field has focused on incorporating various baryonic feedback processes into the simulations. However, there is still an extensive debate in the literature on whether these corrections are sufficient to resolve the galactic scale discrepancies, or if we need to consider modifications of the CDM paradigm. This last possibility is not as exotic as it may sounds. In fact, it certainly seems plausible to postulate that dark matter, which contributes four times as much to the energy density as baryonic matter, exists as part of an entire dark sector with a rich structure analogous the Standard Model (SM) of particle physics.

Over the last three decades, a plethora of models of dark matter have been proposed, of which a long list directly addresses the controversy of the ΛCDM model at small scales [2]. A full treatment of each of these models would require an entire set of lectures and it is thus beyond the scope of this paper. Here, I will discuss in generality two possible departures from the CDM paradigm and their implications for the physics at the galactic scale. More precisely, I will consider the case where DM has: (1) a non-negligible self-scattering cross section (*self-interacting* DM) and (2) that it can exists in two different phases of matter (*superfluid* DM).

II. SMALL-SCALE CHALLENGES

A. The Cusp-Core problem

ΛCDM simulations that include only cold dark matter predict cuspy density profiles. The best fitted density profile from these N-body simulations is known as the Navarro-Frank-White (NFW) profile and it takes the form [6]:

$$\rho(r) = \frac{\rho_0}{(r/r_s)(1 + r/r_s)^2}, \tag{1}$$

where ρ_0 and the scale radius r_s are parameters which vary from halo to halo. From eq.1 we can see that as $r \to 0$, the profile becomes a steep power-law with $\rho(r) \propto r^{-\gamma}$ and $\gamma \simeq 1$[1].

This prediction seems to disagree with observations, specifically for dwarf galaxies[2] which favor a constant-density core with $\gamma \simeq 0.5$ [2]. The observational evidence of this tension is vast and has not alleviated over the past two decades. In [7], they derive inner DM halo density profiles for a sample of 165 low-mass galaxies using their rotation curves and measure median inner slope $\gamma = 0.22 \pm 0.08$[7]. Similarly, high resolution observations of the dwarf irregular galaxies NGC 6822[8] and NGC 3741[9] favor a core rather than the cuspy NFW profile. Finally, the high-resolution rotation curves of 26 dwarfs from LITTLE THINGS yield a mean slope $\gamma = 0.32 \pm 0.24$ [5] which is consistent with the general picture of a core-like inner profile.

Fig.1 summarizes this discrepancy between simulations and observations. The blue and red dots represent the observed rotation curves for two galaxy with same asymptotic velocity $V_{\max} \simeq 40\mathrm{km} \cdot \mathrm{s}^{-1}$ from [5], compared with the black dashed line which represents the velocity curve predicted for an NFW CDM halo with equivalent $V_{\max}$. The two profiles depart significantly for $r \lesssim 3\mathrm{kpc}$ where the observed rotation curves rise much more slowly than the ΛCDM expectations.

In the context of ΛCDM, possible solutions to the core-cusp problem must be a result of either (1) neglected physical processes that affect mainly the baryonic matter, or (2) systematic effects from observations.

Many advanced hydrodynamic simulations have shown that it is possible for baryonic feedback to erase the central cusps predicted by CDM-only simulations [1, 10–12]. In short, these feedback processes from stars and

[2] Dwarf galaxies are low-mass ($M_* \lesssim 10^9 M_\odot$) dark matter dominated galaxies and thus provide a crucial tool for testing the CDM paradigm.

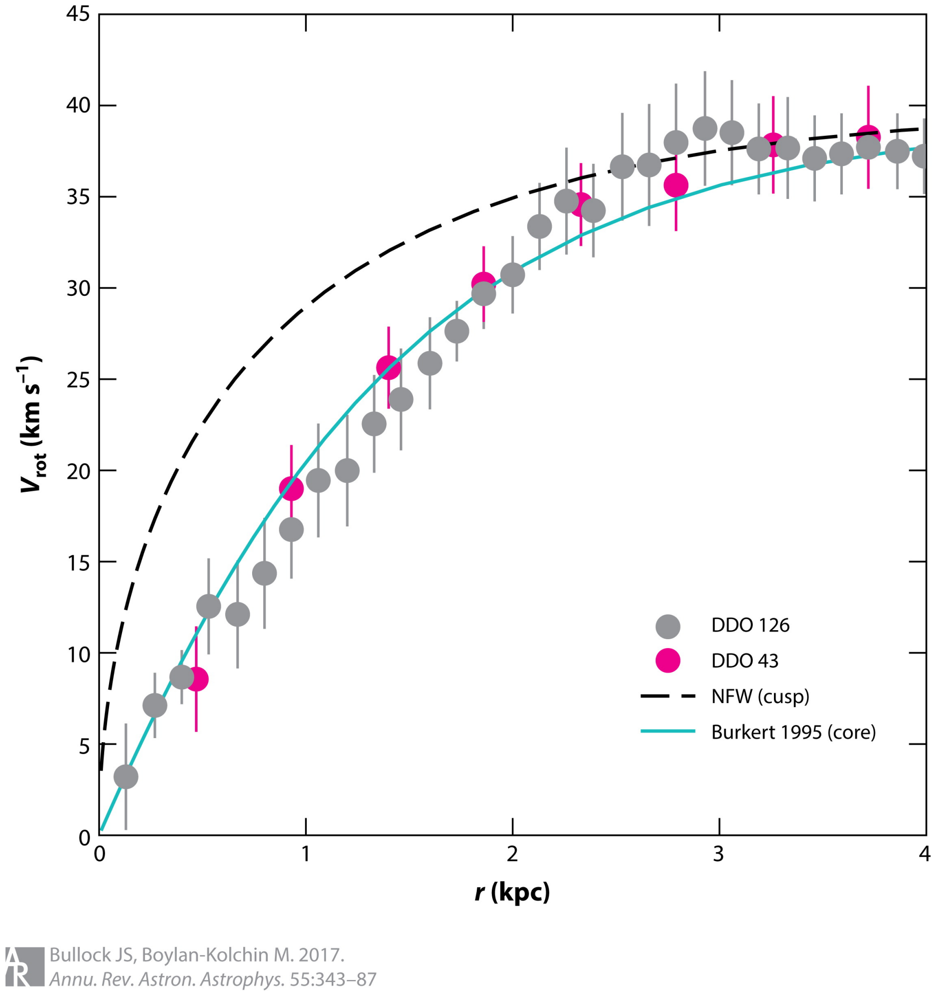

FIG. 1: The Cusp-Core problem. The dashed line shows the ΛCDM expectation (NFW profile) for a typical rotation curve with $V_{\max} = 40\text{km}\cdot\text{s}^{-1}$. This rotation curve rises quickly which reflects the central density profile behaviour $\rho(r) \propto 1/r$. The data points in pink and gray show the rotation curves of two example galaxies of this size from [5], which are more slowly rising and better fit by a density profile with a constant density core. Credit: [1]

active galactic nuclei result in gas density fluctuations which can contribute to heating DM halos and decrease their density at the center, forming more realistic core-like density profiles. The most popular mechanisms rely on supernova and stellar winds which produce an energy feedback that can significantly alter the shape of the galaxy by forcing the gas and DM particles to move outwards [2]. That said, this process is strongly dependent on the mass of the stars formed, and has been shown to be effective only for dwarf galaxies with masses $M_* \gtrsim 10^6 M_\odot$ [1]. In fact, if too few stars are formed, there will not be enough energy in the supernovae to alter the halo density shape, and the resultant profile will resemble the NFW from CDM-only simulations. When galaxies form enough stars ($M_* \sim 10^{8-9} M_\odot$), there will be enough supernovae energy to redistribute dark matter and create significant cores. On the other hand of the extreme, if too many baryons end up in stars, the excess central mass can compensate and drag dark matter back in [1].

As previously mentioned, the other possible solution for the core-cusp problem arises from systematic effects due to observational limits. These include non-circular motions and pointing effects hiding the signature of a cusp in the innermost region of the galaxies [2]. The importance of this effect is still an area of active research. In [13] they use a sample of galaxies from THINGS and estimate the non-circular motions to be of the order of few kilometres per seconds which would be too small to overlook the presence of a cusp[2]. On the other hand, high-resolution simulations suggest that non-circular motions can substantially affect the observed rotation curves and that correcting for these motions in observations is not trivial [14], leaving the relevance of this effect still up to debate.

B. The Missing Satellites Problem

The missing satellites problem (MSP) refers to the discrepancy between the number of predicted subhalos and the number of observed satellite galaxies within the Local Group. CDM-only simulations of the Milky Way (MW) predict 10^{2-3} subhalos large enough to host galaxies, a number in sharp contrast with the ~ 50 known satellites with $M_* > 300 M_\odot$ within 300kpc of the MW [1]. Even though future surveys will certainly bring the number of detected ultra-faint dwarf galaxies up, it is very unlikely that this number will reach the thousands predicted by the CDM simulations. The rich substructure of CDM halos is a result of the the high-density cores of smaller merging halos that survive the hierarchical assembly process. Thus, to resolve this problem within the context of ΛCDM, we must posit that galaxy formation becomes increasingly inefficient as the halo mass drops [1]. In other words, these halos are either not massive enough to gravitationally attract cold gas to form stars, and/or there are physical phenomena undergoing within these subhalos that prevent the formation of luminous stars. A simple mechanism that can achieve this suppression is again baryonic feedback, caused for example by ionizing photons that heat up colder gas, supernovae explosions that eject colder gas outside the galaxy and stellar winds. In addition, a better understanding of the sensitivity of telescopes that observe these dwarf galaxies, and thus of the completeness of the surveys, is necessary to bridge the gap between the two galaxy counts [2]. In fact, it is important to emphasize that the observed galaxy count is just a lower bound on the population of MW satellites, since only a fraction of the MW's virial volume has been surveyed [15]. In short, resolving the MSP requires that the completeness-corrected galaxy count matches the predicted luminous satellite abundance [16].

Modelling the relationship between the galaxies and their host DM halo taking these phenomena into account is not a simple task and it is generally done by matching the cumulative distribution of an observed property of galaxies with the predicted cumulative distribution of the mass of their DM halos [2]. This technique is called Abundance Matching (AM) and it stands on the assumption that galaxies and DM halos are related in a one-to-one way, with the most massive DM halos hosting the most massive galaxies [1]. Typically modelling via AM is done with the stellar mass. Many models of this kind have been put forward (Behroozi[17], Garrison-Kimmel[18, 19], Moster[20] and Brook[21]) and they adopt different galaxy data sets and the dark matter halo mass function from different N-body simulations [2]. For $M_* > 10^8 M_\odot$ the models are consistent with one another, while they differ at smaller stellar masses, where the incompleteness of the surveys and the increased stochasticity of the star formation mechanism in the models makes the estimation of the stellar mass uncertain and, consequently, the AM relation less constrained [2]. In [22], the above mentioned AM models, both with and without the inclusion of reionization effects, have been shown to resolve the MW satellite problem for $M_* \gtrsim 4.5 \cdot 10^5 M_\odot$. In [16], by including the large completeness corrections given the new discoveries of ultra-faint dwarfs by the Sloan Digital Sky Survey, they claim to solve the MSP down to masses $M_* \sim 10^3 M_\odot$ and that if anything, there may be a *too many satellites* problem. Another strategy is to abundance match with the mean star formation rate, averaged over the time when a galaxy was forming stars. This is done to avoid the intrinsic scatter of stellar mass -$M_{\rm halo}$ relation, due to several processes that can occur within the galaxy (e.g. suppression of star formation due to either infall of the galaxy towards a larger galaxy or tidal stripping [2]). This technique is used in [23] where they find that the missing satellite problem disappears for masses $M_* > 10^9 M_\odot$.

Given all of these considerations, the lack of MW satellites is certainly not as severe as originally thought, with many scientists in the field that would go as far as claiming that this discrepancy is simply non-existent. In the future, an important avenue will be to push these comparisons down to the ultra-faint regime, where the uncertainty in the AM models used in these analysis is still quite large [1].

It is worth pointing out that the solutions proposed to the missing satellites problem are inconsistent with another challenge of the ΛCDM at small scales, the so called *too big too fail* (TBTF) problem. In short, the AM technique used to resolve the MSP is based on the assumption that the known MW satellites should reside in the the largest DM subhalos. The comparison of the observed central masses to ΛCDM simulations revealed that the most massive CDM subhalos were systematically too centrally dense to host the bright MW satellites [3]. This density discrepancy can be resolved by assuming that the most massive subhalos are dark and the brightest satellite galaxies reside in subhalos that are ~ 5 less massive [2]. Clearly, this scenario contradicts the assumption of AM and generates a new challenge for ΛCDM on its own. The largest DM subhalos are characterized by the deepest potential wells and thus are expected to be able to retain gas and form stars[3]. While in principle separate, the TBTF and core-cusp problems are naturally connected if low-mass galaxies generically have DM cores, as this would reduce their central densities compared to CDM expectations [1]. For the purpose of this paper, I will therefore not treat the two issues separately.

C. Baryonic scaling relations

One of the most puzzling feature of galaxy phenomenology in the context of ΛCDM are the tight correlations between observed kinematic quantities and quantities associated to the baryonic component, even within systems that are dark matter dominated [1]. The most striking example of this is the baryonic Tully-Fisher relation (BTFR), plotted in fig.2, which shows a remarkably tight connection between the total baryonic mass of a galaxy M_b and the asymptotic flat velocity of its rotation curve V_c [2]:

$$M_b = AV_c^4 \tag{2}$$

where the normalization constant is $A = 47 \pm 6\ M_\odot \mathrm{km}^{-4}\mathrm{s}^4$ [31]. A generalization of the baryonic Tully-Fisher relation is known as the radial acceleration relation (RAR) which shows a tight correlation between the observed centripetal acceleration traced by rotation curves ($g_{\mathrm{obs}} = V^2(r)/r$) and the Newtonian acceleration $g_{\mathrm{bar}}(r)$, due to baryonic matter alone [32]:

$$g_{\mathrm{obs}}(r) = \frac{g_{\mathrm{bar}}(r)}{1 - \exp\left(-\sqrt{\frac{g_{\mathrm{bar}}(r)}{g_\dagger}}\right)}, \tag{3}$$

where $g_\dagger = (1.20 \pm 0.02 \pm 0.24) \cdot 10^{10} \mathrm{m} \cdot \mathrm{s}^{-2}$ [32, 33]. The crucial feature of these correlations is their small intrinsic scatter. This implies that irrespective of the merging history of the galaxy, its DM halo, that represents $\sim 90\%$ of the galaxy mass and thus sets its dynamical properties, seems to adjust its properties to those of the luminous matter, that only accounts for $\sim 10\%$ of the galaxy mass [2].

ΛCDM hydrodynamical simulations have already been shown to predict the same normalization and slope of the RAR and BTFR [31, 34–36]. That said, the systems simulated did not include faint dwarf galaxies, which are dark

[3] In other words, they are too big to fail!

matter dominated throughout, and most importantly they show a considerably larger scatter than observations. Explaining the remarkably small level of scatter of these empirical relations is in fact the real challenge for ΛCDM. As mentioned above, in the CDM scenario, cosmic structures form hierarchically through stochastic mergers, and we would expect a large scatter, mirroring the merging history of each galaxy. It is important to emphasize that for a fixed asymptotic velocity $V_{\max}$, galaxies demonstrate a huge diversity in their central densities. Surprisingly, this diversity seems to be correlated with the baryonic content in such a way as to drive the tight relation seen in fig.2 [1].

Intriguingly, the RAR was predicted 40 years ago predicted by Milgrom [37] with MOdified Newtonian Dynamics (MOND). MOND proposes to completely replace dark matter with a modification of the Newtonian force law in the regime of very low acceleration. Specifically, the MOND law states that the gravitational acceleration a is related to the baryonic acceleration a_b[4] via [37]:

$$a = \begin{cases} a_{\rm b} & a_{\rm b} \gg a_0 \\ \sqrt{a_{\rm b} a_0} & a_{\rm b} \ll a_0 \end{cases} , \tag{4}$$

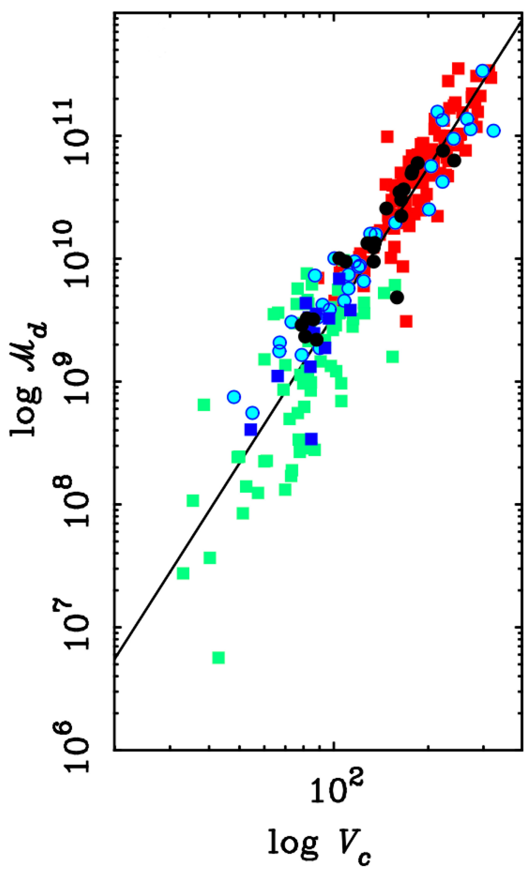

FIG. 2: The Baryonic Tully Fisher Relation. The figure plots the total baryonic disk mass of galaxies against the rotation velocity V_c. Circles and squares represent the data derived from [24] (red), [25] (black), [26, 27] (green), [28] (light blue) and [29] (dark blue). The black solid line shows an unweighted linear fit. Credit: [30]

[4] the Newtonian acceleration due to ordinary matter alone.

where a_0 is a characteristic acceleration scale, whose best fit value is of the order the speed of light times the Hubble constant H_0 [38, 39]:

$$a_0 \simeq \frac{1}{6} c H_0 \simeq 1.2 \times 10^{-8} \text{ cm/s}^2. \tag{5}$$

MOND does extremely well at fitting detailed galactic rotation curves [38, 40]. In fact, the BTFR and RAR can be thought as an exact consequence of the force law in eq.4. The empirical success of MOND is only limited to galaxies. At cosmological scales, in order to reproduce the observed temperature profile of galaxy clusters and the CMB power spectrum, one must introduce a significant dark matter component [41–43]. Despite of all of this, the success of the MOND law at fitting galactic properties is unequivocal. This acceleration scale a_0 is clearly in the data. Therefore, regardless of the nature of the dark matter that one may postulate, its density profile in galaxies must at the end of the day conform to MOND [39].

III. BEYOND THE COLD DARK MATTER PARADIGM

A. Self Interacting Dark Matter

In the previous section I presented the discrepancies of the CDM paradigm at galactic scales and I showed how the inclusion of baryonic feedback processes can alleviate at least some of these inconsistencies. However, the significance of these effects is strongly model-dependent and the community has not reached a consensus over how baryon dynamics actually affect halo properties in reality. Thus, it remains an intriguing possibility that these discrepancies are a result of a breakdown of the CDM paradigm at galactic scales. In other words, we may postulate extensions of CDM which reproduce a cold, collisionless fluid on large scales, while departing significantly from this picture on small scales.

As a first added level of complexity, we can consider cold DM particles with a non-neglegible scattering cross section per unit mass ($\sigma/m \equiv \tilde{\sigma}$). This model is called self-interacting cold dark matter (SIDM) and it was first introduced by Spergel and Steinhardt [44] as a mechanism for smoothing out substructure on small scales without spoiling the successes of the CDM paradigm at cosmological scales.

The basic idea is that: at large length scales, the rate of collisions becomes negligible due to the smaller density and we recover an effectively collisionless CDM; while at smaller length scales, we require that the SIDM particles undergo a non-negligible number of collisions, in such a way to alter the halo evolution and alleviate the galactic scale discrepancies of ΛCDM. This condition imposes an upper bound on the SIDM local mean free path λ. In fact, if λ is much longer than $\sim$ 1Mpc, the typical DM particle would not experience any interaction as it moves through the halo [44]. More precisely, at the solar radius in a Milky Way-like galaxy, for a typical SIDM particle moving at $v_0 \approx 220 \text{km} \cdot \text{s}^{-1}$ we require at least 1 collision per Hubble time[46], thus[5]:

$$\lambda \lesssim \frac{v_0}{H_0} = 3.14 \text{ Mpc}. \tag{6}$$

On the other hand, a lower bound on λ is set by requiring that, as observed, clusters are triaxial rather than spherical on large scales [46]. In fact, collisions tend to make velocity distribution isotropic such that, for a λ much smaller than 1kpc, the halos could not be triaxial but only spherical or elliptical, depending on the significance of

[5] Here and for the rest of the paper, I take $H_0 = 70 \text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$.

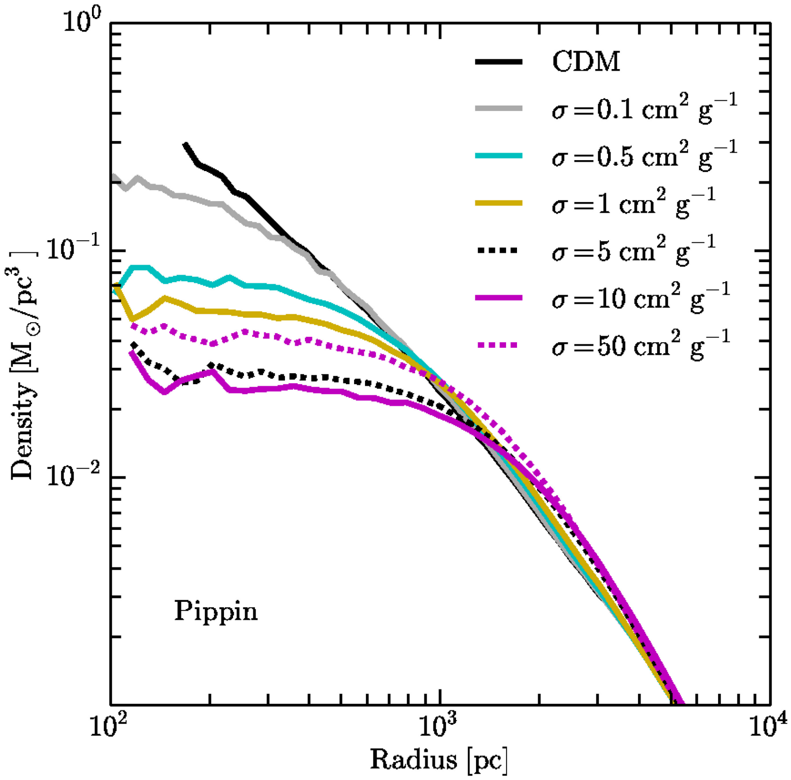

FIG. 3: Density profiles of SIDM. The figure shows the inner density profile of a DM halo with $M_{\rm halo} = 0.9 \cdot 10^{10} M_\odot$ in collisionless CDM and in SIDM for different values of $\tilde{\sigma}$. Credit: [45].

the angular momentum. More precisely, we require that there is less than one collision per Hubble time at the viral radius of a DM halo [46] [6]:

$$\lambda \gtrsim \frac{v_0}{H_0} \cdot \frac{200\rho_{m,0}}{\rho_{\rm MW,0}} = 2.6 \text{ kpc}, \tag{7}$$

where $\rho_{m,0} = 1.79 \cdot 10^{-6}$ GeV $\cdot$ cm^{-3}, $\rho_{\rm MW,0} = 0.39$ GeV $\cdot$ cm^{-3} is the local DM density [47] and v_0 is taken to be approximately the same as at the solar radius[7]. The cross section per unit mass is related to the mean free path by the equation:

$$\tilde{\sigma} = \frac{1}{\lambda(r) \cdot \rho(r)}. \tag{8}$$

By setting the local density of DM to $\rho_{\rm MW,0} = 0.39$ GeV $\cdot$ cm^{-3} as done above, we derive the corresponding range of allowed $\tilde{\sigma}$ for SIDM:

$$0.1 \text{ cm}^2 \cdot \text{g}^{-1} \lesssim \tilde{\sigma} \lesssim 200 \text{ cm}^2 \cdot \text{g}^{-1} \tag{9}$$

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[6] Note that the viral radius corresponds to the radius at which the density is approximately 200 times the mean matter density of the universe $\rho_{m,0} \equiv \Omega_m \cdot \rho_{c,0}$ where $\rho_{c,0} \equiv 3H_0^2/(8\pi G)$ is the critical density.

[7] Here I take the fiducial values of $\Omega_m = 0.3$ and $\rho_{c,0} = 5.96 \cdot 10^{-6}$ GeV $\cdot$ cm^{-3}, in accordance with the latest results from the Planck Collaboration [48].

Postulating a dark matter particle with a scattering cross section between the two extremal values will not affect observations on large scales. In fact, for distances r beyond the viral radius of a DM halo, we have $\lambda(r)/r \ggg 1$ which implies SIDM can be treated effectively as an optically thin, collisionless gas. However, at smaller radii SIDM becomes optically thick ($\lambda(r)/r \lll 1$) and has multiple collisions within an orbit. Infalling dark matter is thus scattered before reaching the center of the galaxy so that the orbit distribution is isotropic rather than radial. The collisions heat up the low entropy material that would usually collapse to form a cusp and produce a shallower density profile [44]. This mechanism has clearly the potential to resolve the core-cusp problem of ΛCDM without the recourse to baryonic feedback processes (see fig.3). In [45], SIDM-only simulations are shown to resolve the core-cusp problem if $\tilde{\sigma} \gtrsim 0.5\ \mathrm{cm}^2 \cdot \mathrm{g}^{-1}$ for galactic halos of $M_{\mathrm{halo}} \sim 10^{11} M_\odot$. However, observational constraints on cluster scales, i.e. for $M_{\mathrm{halo}} \sim 10^{14} M_\odot$, require $\tilde{\sigma} \sim 0.1\ \mathrm{cm}^2 \cdot \mathrm{g}^{-1}$ [49]. This difference in the value of $\tilde{\sigma}$ at galaxy and cluster scales calls for a more complicated functional form for the cross section as a function of velocity. More precisely, $\tilde{\sigma}(v)$ must decrease as the rms speed of DM particles involved in the scattering rises from the scale of dwarfs ($v \sim 10\ \mathrm{km} \cdot \mathrm{s}^{-1}$) to galaxy clusters ($v \sim 1000\ \mathrm{km} \cdot \mathrm{s}^{-1}$) [1].

An additional aspect to consider in SIDM models are the interactions of DM particles within subhalos with those within the main halo. Since subhalos have much smaller velocity dispersions, most two-body scatterings between the satellites and the halo DM particles will lead to the ejection of both particles from the subhalos, gradually destroying all the substructure within the inner portion of the main halo [46]. This is precisely the mechanism that would be needed to resolve the missing satellite problem. Simulations of velocity dependent SIDM models have been shown to successfully resolve this discrepancy [2, 50]. However, this class of models is still largely unconstrained.

B. Superfluid Dark Matter

In the previous section we saw that by simply introducing a non-negligible self-interacting cross section, we were able to significantly alter the subhalos evolution, in such a way to alleviate some of galactic-scale discrepancies presented in sec.II. However, this simple version of SIDM, as originally envisioned by Steinhardt and Spergel, tells us very little about the nature of the tight baryonic scaling relations of the galaxy rotation curves. To resolve this discrepancy, we are thus motivated to add another level of complexity to our dark matter model. As emphasized in sec.II C, we know that in order to reproduce the RAR and BTFR, the DM density profile in galaxies must conform to MOND. With this in mind, Berezhiani and Khoury [44] proposed a unified framework for the CDM and MOND phenomena based on the physics of superfluidity. In this approach, the CDM and MOND components represent different phases of a single underlying substance. Specifically, at cosmological scales, we can treat dark matter as a pressureless, non-interacting fluid; at galactic scales, dark matter forms a superfluid with coherence length of the galactic size. Ignoring the interactions between the DM particles, the condition for the onset of superfluidity simply amounts to imposing that their de Broglie wavelength $\lambda_{dB} \sim 1/(mv)$ is larger than the inter-particle separation $l \sim (m/\rho)^{1/3}$ at the galactic scale [51]. This translates into an upper bound on the mass of DM particles:

$$m \lesssim (\rho/v^3)^{1/4} \tag{10}$$

Substituting the local DM density ($\rho_{\mathrm{MW},0} = 0.39\ \mathrm{GeV} \cdot \mathrm{cm}^{-3}$) and velocity dispersion in the Milky way ($v \simeq 100\mathrm{km} \cdot \mathrm{s}^{-1}$), we get $m \lesssim 6\mathrm{eV}$. Another requirement for Bose-Einstein condensation (BEC) is that DM thermalizes within galaxies. This amounts to demanding that the interaction rate between DM particles is larger than the galactic dynamical time, which corresponds precisely to the lower bound on $\tilde{\sigma}$ derived in sec.III A [51]. In short, superfluidity requires the DM particles to be sufficiently light ($m \sim eV$) and have strong self-interactions ($\tilde{\sigma} \sim 0.1\ \mathrm{cm}^2 \cdot \mathrm{g}^{-1}$). Again ignoring interactions, the critical temperature for DM superfluidity can be estimated to be

$T_c \sim mK$, which is intriguingly comparable to known critical temperatures for cold atom gases [51].

Under these conditions, galaxies are almost entirely condensed. Therefore, at this scale, DM particles are better described as coeherent excitations: phonons and massive quasi-particles. By a convenient choice of the phonon action and of its coupling term to baryonic matter, the DM superfluid can give rise to a long-range phonon-mediated force between ordinary matter particles which precisely reproduces the characteristic acceleration scale of MOND. Specifically, Berezhiani and Khoury conjecture that superfluid DM phonons are described by scalar field θ with the following effective action [51] [8]:

$$\mathcal{L} = P(X) + \mathcal{L}_{\rm int}, \tag{11}$$

$$P(X) = \frac{2\Lambda(2m)^{3/2}}{3} X\sqrt{|X|}, \quad \mathcal{L}_{\rm int} = -\alpha \frac{\Lambda}{M_{\rm pl}} \theta \rho_b; \tag{12}$$

where $X \equiv \dot{\theta} - m\Phi - \frac{(\vec{\nabla}\theta)^2}{2m}$, Φ is the external gravitational potential and α is the coupling constant for the interaction between phonons and the the baryon mass density ρ_b . To reproduce the MONDian force, α is fixed in terms of Λ via the critical acceleration a_0:

$$\alpha^{3/2}\Lambda = \sqrt{a_0 M_{\rm pl}} \simeq 0.8 \text{ meV}. \tag{13}$$

As a result of this, a test particle orbiting the galaxy would be subject to two forces: the (Newtonian) gravitational force (from both the baryons and the DM particles) and the phonon-mediated force ($a_\pi = \sqrt{a_0 a_b}$). In this way, this model provides a physical explanation for the peculiar baryonic scaling relations described in sec.II C, simply as an emergent phenomenon from the coherence of the underlying superfluid substrate of dark matter at galactic scales [51].

Finally, it is worth emphasizing the natural distinction in the behaviour of DM particles at cluster versus galactic scales that arises in this framework. As a general result from thermodynamics, a superfluid at finite temperature (and below the critical temperature) is best described phenomenologically as a mixture of two fluids: (1) the superfluid, which by definition has vanishing viscosity and carries no entropy; (2) the normal viscous component carrying the entropy [53]. The fraction of particles in the superfluid state decreases with increasing temperature. Thus, in galaxy clusters, which have a higher velocity dispersion and correspondingly a higher DM temperature, only a small negligible fraction of DM particles is in the superfluid state, and we recover the CDM picture.

IV. CONCLUSIONS

In this paper I presented the small-scale challenges of the CDM paradigm and the possible mechanisms that would be necessary to resolve them, both within and beyond the ΛCDM framework. Overall, these inconsistencies at galactic scales are not as severe as originally thought. The development of advanced N-body simulations that include the effects of baryonic feedback processes, combined with the enhancement of the sensitivity in galaxy surveys, have been shown to alleviate and in some cases fully resolve the tensions of the CDM model. In the future, an important avenue will be to push the comparisons between simulations and observations down to the ultra-faint regime, where the systems are fully dark matter dominated and feedback processes are expected to play a negligible role.

[8] Generally, for a fundamental scalar field, the fractional power (3/2) of X in the effective action would seem a bit strange and unphysical. However, as a theory of phonons, it is not uncommon to see fractional powers in cold atom systems. An example of this is the Unitary Fermi Gas (UFG) [52], which has generated much excitement recently in the cold atom community [53].

Of the challenges of the CDM paradigm presented here, the low scatter from the baryonic scaling relations remains the most puzzling. In fact, while hydrodynamical ΛCDM simulations have been shown to predict the same slope and normalization, they cannot reproduce the small scatter seen in the observations. Perhaps, this is the clearest hint that a breakdown of the CDM paradigm is necessary at these scales. An example of this is the superfluid DM model postulated by Berezhiani and Khoury, which offers a natural explanation for the emergence of a MONDian-like force from the coherence of the underlying dark matter medium.

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